# The mathematics of learning as integration 

Rodolfo Alvarez


#### Abstract

The purpose of this investigation is proposing a subconcept of mathematics for the explanation of learning, conceived as integration of new elements to previous knowledge. In this way, previous set of knowledge changes as a whole every time a new element is added, thus generating a deeper understanding in general terms. The concept used to explain learning in this case, is mathematical function, adding a new dimension each time a minimal value is added to the function.


Index Terms- dimension, function, integration, learning.

## 1 Introduction

LEARNING is a concept allowing for diverse means of exsplanation. One of those means is mathematical language, through which answers aiming at a higher level of precision can be reached. In that sense, it is one of the valid options to explore the concept of learning.

In this article we explore the mathematics behind a conception of learning in which a minimal value is added to previous knowledge, forming the knowledge currently had. The approach used seems to be retrospective at first sight but a second look at it, will show it hides a progressive explanation for learning phenomenon.

## 2 LITERATURE REVIEW

### 2.1 Learning as integration

Fink (2013) partially defines learning as integration in terms of connection. If we go a little back in time, we have Selnes \& and Sallis definition of learning which is "integration into existing memory" (1999).

Both definitions are compatible with what we try to explain. Combining both of them allows for a more powerful conceptualization of learning as integration. In that sense, we can define learning as "connective integration of a new element into existing memory" (Fink, 2013; Selnes \& and Sallis, 1999). We understand memory as a synonym of previous knowledge in this case.

### 2.2 Mathematical function

A mathematical function is a rule taking a value and producing another value (input/output relation) (Beliakov et. al, 2007). It can be defined in slightly more rudimentary terms as a rule giving value of a dependent variable corresponding to specified values of an independent variable (Mortimer, 2013).

## 3 DISCUSSION

### 3.1 Adding dimensions

Let us suppose we try to find a formula, a function in this case, to represent the addition of a new element of knowledge into somebody's mind, or somebody's previous knowledge. After a while thinking of a mathematical means to represent the idea of learning entering the mind, we can come up with the following function:
$\quad x=y+1$
, in which $x$ stands for current knowledge, $y$ stands for previous knowledge, and 1 stands for new knowledge acquired. In that sense, previous knowledge (y) plus new knowledge (1) produce current knowledge (x).

However, this does not seem to be enough, since we want to give an account not only of the learning process in one computation. We also aim at exploring the historical trace of previous learning to see how it works together with new knowledge acquisition.

For the purpose of what has already been mentioned, we can add a new function to the discussion, which is the following:

$$
y=z+1
$$

In this case, $y$ value (from the formula $x=y+1$ ) meaning previous knowledge, has its own equivalent, which is $z+1$. We know what $y$ is but another question arises: What is $z+1$ ? We know it equals y therefore we know $z+1$ means previous knowledge compared to $x$, which is current knowledge.

However, to really know what $z+1$ means, we have to analyze both elements one by one. First, z is previous knowledge before $y$, meaning $y$ was current knowledge at some point. To make y possible, we had $z$ in the first place, which is previous knowledge before previous knowledge. When we added 1 to z $(z+1)$, we represent the new knowledge once acquired to produce y (once current knowledge).

If we go on with the process, we can ask what z is, and then we have the following formula:

$$
\mathrm{z}=\mathrm{z}^{\prime}+1
$$

- Rodolfo Alvarez, Universidad de los Lagos, Chile.

E-mail: rodolfo.alvarez@ulagos.cl
, in which z value (from the formula $\mathrm{y}=\mathrm{z}+1$ ) has its own equivalent, which is $z^{\prime}+1$. Just like previous function, we know what z is but now we ask: What is $\mathrm{z}^{\prime}+1$ ? We know it equals $z$ therefore we know $z^{\prime}+1$ is previous knowledge before previous knowledge compared to y , which was current knowledge at some point.

However, to really know what $\mathrm{z}^{\prime}+1$ means, we have to analyze both elements one by one. First, $z^{\prime}$ is previous knowledge before z , meaning z was current knowledge at some point. To make $z$ possible, we had $z^{\prime}$ in the first place, which is previous knowledge before previous knowledge before previous knowledge. . When we added 1 to $z^{\prime}\left(z^{\prime}+1\right)$, we represent the new knowledge once acquired to produce $z$ (once current knowledge).

We will add a new dimension to the discussion to have a full idea of the proposal. However, the process can be endless since we can add new dimensions on and on. The function to be presented is the following:

$$
z^{\prime}=z^{\prime \prime}+1
$$

, in which $z^{\prime}$ value (from the formula $z=z^{\prime}+1$ ) has its own equivalent, which is $z^{\prime \prime}+1$. Just like previous function, we know what $z^{\prime}$ is but now we ask: What is $z^{\prime \prime}+1$ ? We know it equals $z^{\prime}$ therefore we know $z^{\prime \prime}+1$ is previous knowledge before previous knowledge compared to $z^{\prime}$, which was current knowledge at some point.

However, to really know what $z^{\prime \prime}+1$ means, we have to analyze both elements one by one. First, $z^{\prime \prime}$ is previous knowledge before $z^{\prime}$, meaning $z^{\prime \prime}$ was current knowledge at some point. To make $z^{\prime}$ possible, we had $z^{\prime \prime}$ in the first place, which is previous knowledge before previous knowledge before previous knowledge before previous knowledge. When we added 1 to $z^{\prime \prime}\left(z^{\prime \prime}+1\right)$, we represent the new knowledge once acquired to produce $z^{\prime}$ (once current knowledge).

### 3.2 Learning as integration

Now that we have explained the addition of dimensions representing the process of learning as integration, it is time to present the whole process orderly. First, we have the formula used at the beginning of the discussion:

$$
x=y+1
$$

As we have already said, this function is the initial state of a process that adds dimensions to explain how learning as integration works. One of the aspects involved in the process presented is the fact it represents the historical record of previous knowledges which have been current knowledges at some point.

However, the main point of this research is not exactly giving an account on the historical traces of current knowledge, though explaining it is one of the consequences of this approach, mathematically speaking.

The main purpose of this research is explaining how adding a new element to a given state of knowledge, creates a new set of knowledge, which is ultimately a new dimension.

The concept of dimension is important since in pedagogical
terms, when we learn something and that new knowledge changes our whole knowledge, in a way we upgrade, we are able to be in a higher "dimension" because we understand the world more deeply.

Next formula explains previous function in more detail.

$$
x=(z+1)+1
$$

In this case we know x stands for current knowledge, $\mathrm{z}+1$ stands for $y$ (previous knowledge), and 1 stands for new knowledge acquired. z means previous knowledge before previous knowledge, to which we add 1, forming previous knowledge. Previous knowledge plus 1 form current knowledge.

Next, we have the following function continuing with the process we are explaining:

$$
x=\left(\left(z^{\prime}+1\right)+1\right)+1
$$

In this case we know $x$ stands for current knowledge, $z^{\prime}+1$ stands for z (previous knowledge before previous knowledge), first 1 stands for new knowledge acquired at some point, second 1 stands for new knowledge acquired at some point after first 1, and third 1 stands for new knowledge acquired.
$z^{\prime}$ means previous knowledge before previous knowledge before previous knowledge, to which we add 1, forming previous knowledge before previous knowledge.

Previous knowledge before previous knowledge plus 1 form previous knowledge. Finally, previous knowledge plus 1 form current knowledge.

Next formula is the last function we will present in this article, with its corresponding explanation. However, as it has been said, the whole process could continue endlessly but for the purpose of presentation we end with this one.

$$
x=\left(\left(\left(\left(z^{\prime \prime}+1\right)+1\right)+1\right)+1\right.
$$

In this case we know $x$ stands for current knowledge, $z^{\prime \prime}+1$ stands for $\mathrm{z}^{\prime}$ (previous knowledge before previous knowledge before previous knowledge), first 1 stands for new knowledge acquired at some point, second 1 stands for new knowledge acquired at some point after first 1, third 1 stands for new knowledge acquired at some point, and fourth 1 stands for new knowledge acquired.
$z^{\prime \prime}$ means previous knowledge before previous knowledge before previous knowledge before previous knowledge, to which we add 1, forming previous knowledge before previous knowledge before previous knowledge.

Previous knowledge before previous knowledge before previous knowledge plus 1 form previous knowledge before previous knowledge.

Then, previous knowledge before previous knowledge plus 1 form previous knowledge. Finally, previous knowledge plus 1 form current knowledge.

## 4 CONCLUSION

In this article, we developed the mathematical process by which a new element is added to previous knowledge every time, forming current knowledge, all this in the form of mathematical function.

In that process, a new dimension is added, reflecting not only the historical trace of learning but mainly the effect that new knowledge produces on the learner, allowing for a deeper understanding of the world.

In that sense, learning is integration of a new element every time, creating a new whole with each integration. Each new whole means adding a new dimension.

## References

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